

Calculating Velocity or Displacement From Acceleration Time Histories

By Adrian Lincoln, Prosig

It is quite straightforward to apply “classical” integration techniques to calculate either a velocity time history from an acceleration time history or the corresponding displacement time history from a velocity time history.

The standard method is to calculate the area under the curve of the appropriate trace. If the curve follows a known deterministic function then a numerically exact solution can be found; if it follows a non-deterministic function then an approximate solution can be found by using numerical integration techniques such as rectangular or trapezoidal integration. Measured or digitized data falls in to the latter category. However, if the data contains even a small amount of low frequency or DC offset components then these can often lead to misleading (although numerically correct) results. The problem is not caused by loss of

information inherent in the digitisation process; neither is it due to the effects of amplitude or time quantisation; it is in fact a characteristic of integrated trigonometric functions that their amplitudes increase with decreasing frequency.

Mathematical Background

Consider a single sinusoid of amplitude A and frequency f . This can be represented mathematically as

$$y(t) = A \sin(2\pi ft)$$

The indefinite integral of this waveform is given by

$$\int y(t).dt = c - \frac{A}{2\pi f} \cos(2\pi ft)$$

From this one can see that the amplitude of the oscillatory component is inversely proportional to the frequency: as the frequency increases the amplitude decreases. This can be demonstrated

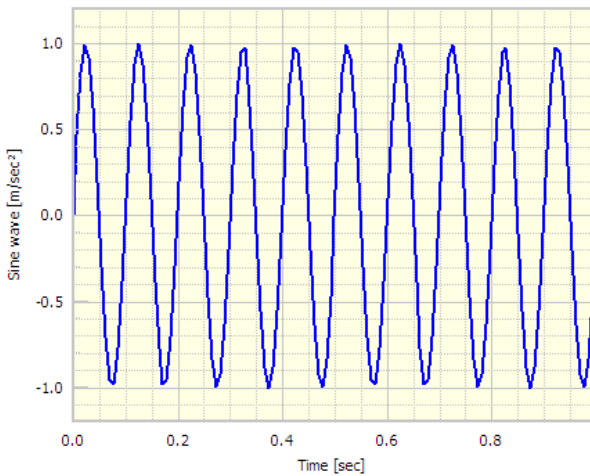


Figure 1: 10Hz Sinewave

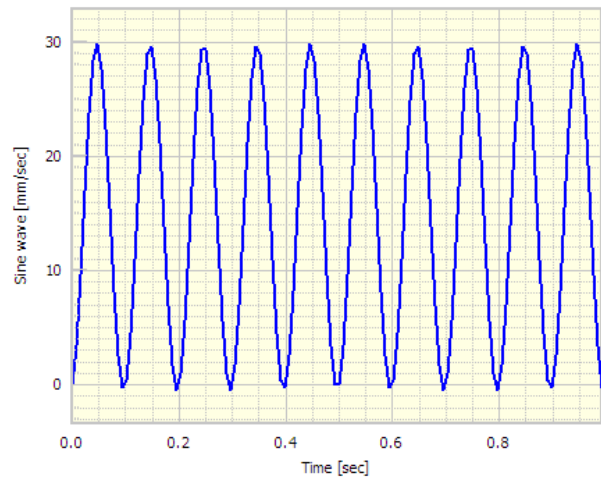


Figure 2: 10Hz Sinewave (Integrated)

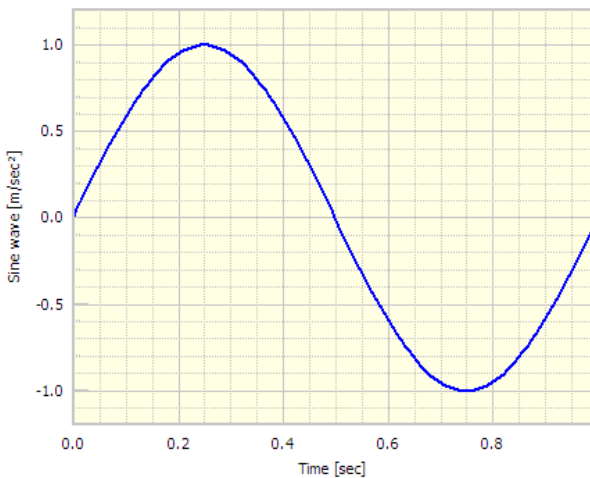


Figure 3: 1Hz Sinewave 1Hz

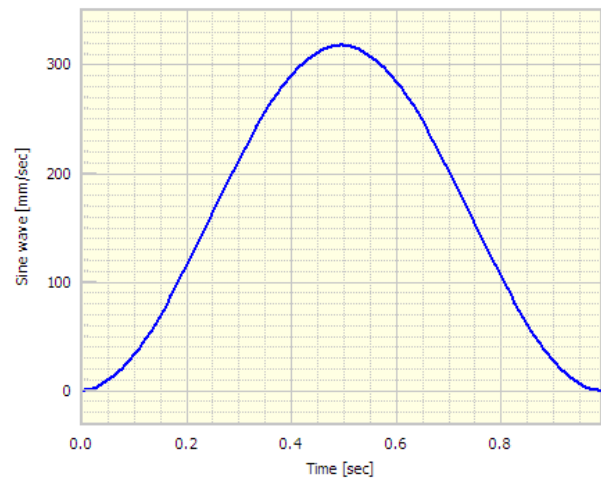


Figure 4: 1Hz Sinewave (Integrated)

graphically as follows. One could generate a digital sinewave of unity amplitude and frequency 10Hz. The resultant waveform and the integral of this waveform are shown in figures 1 and 2.

If one now compares the 10Hz results with those from a sinusoid of the same amplitude but with a lower frequency of 1Hz (as shown in figures 3 and 4) then it is immediately apparent that the integrated 1Hz signal is more than 10 times larger than that of the 10Hz signal.

When the two waveforms are added together the results in figures 5 and 6 are obtained. As can be clearly seen the low frequency behavior dominates the integrated output and the oscillatory characteristic

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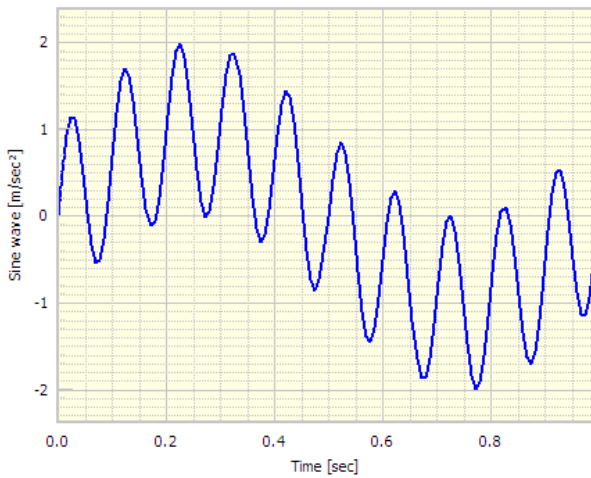


Figure 5: Summed Sinewaves (1Hz + 10Hz)

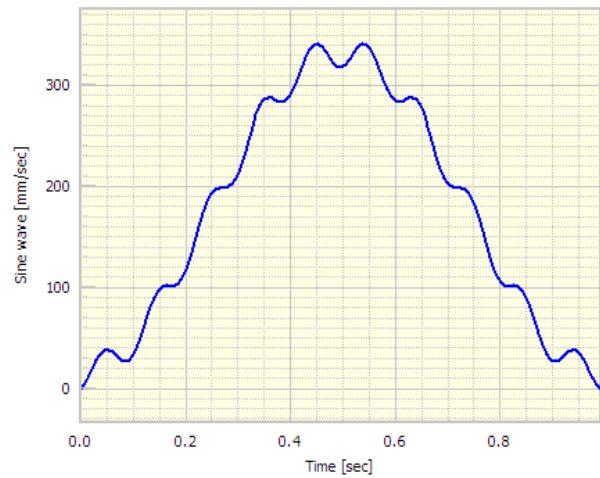


Figure 6: Summed Sinewaves (Integrated)

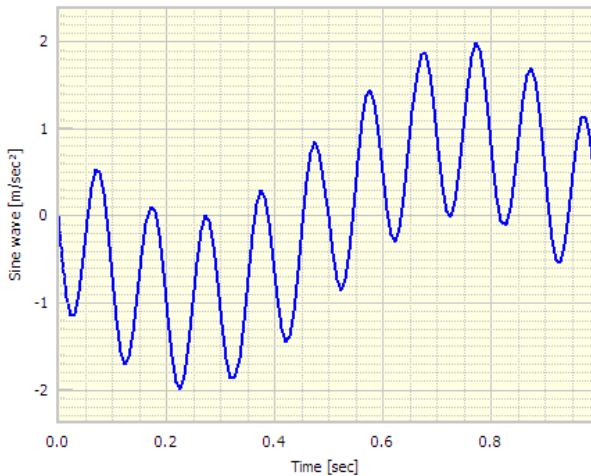


Figure 7: Inverted Summed Sinewaves (1Hz + 10Hz)

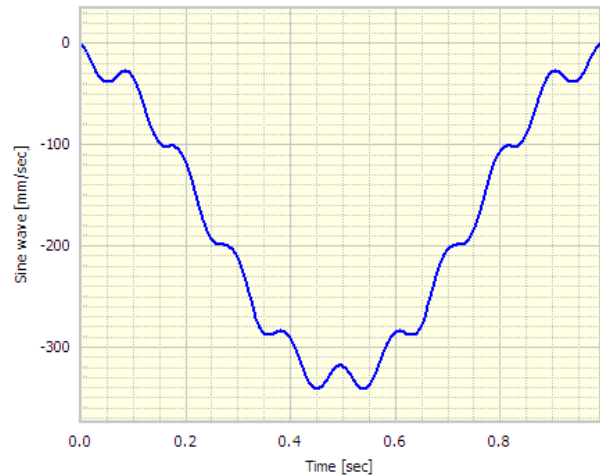


Figure 8: Inverted Summed Sinewaves (Integrated)

of the original waveform is no longer present in the integrated waveform.

Not only is the amplitude of the low frequency component important but the phase is also crucial because it can have a significant effect on the gross shape of the integrated output. An inspection of all the integrated output waveforms shown above reveals that instead of being bipolar like the inputs they are all predominantly positive. This is a consequence of

“With DATS the user has much greater flexibility of choice of cut-off frequency”

the phase of the sinewave input signals being zero. If instead the starting phases were delayed by 180 degrees then the outputs would be predominantly negative as shown in figures 7 and 8.

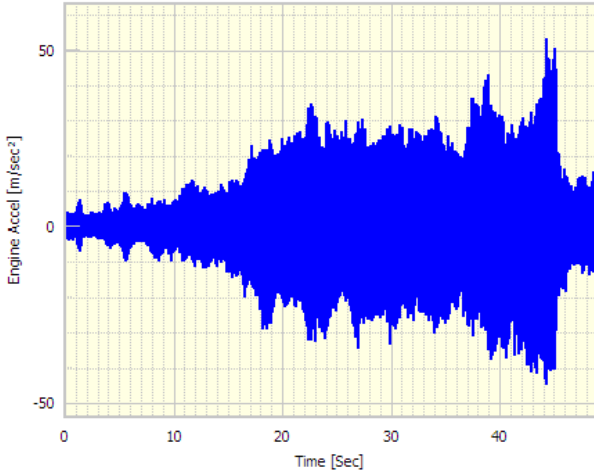


Figure 9: Engine vibration acceleration

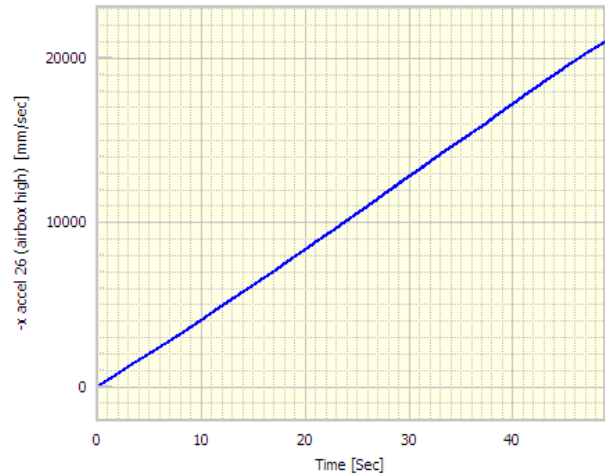


Figure 10: Engine vibration velocity (Integrated)

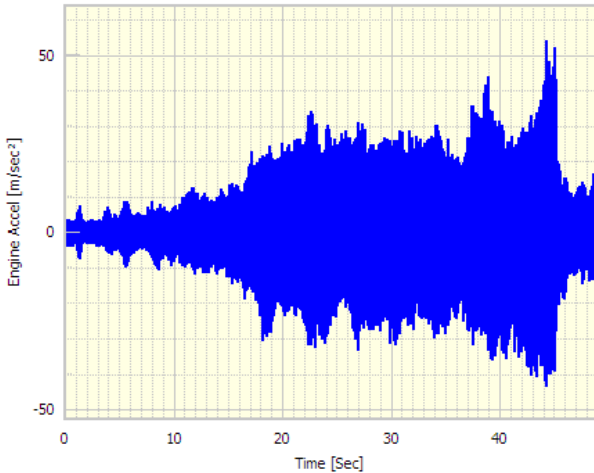


Figure 11: Engine vibration acceleration after high pass filtering

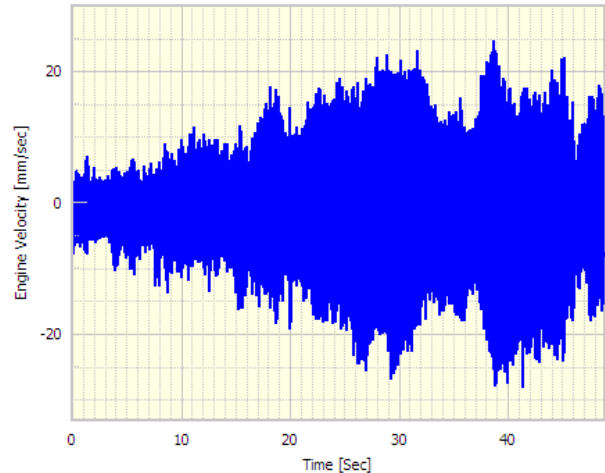


Figure 12: Engine vibration velocity after high pass filtering (Integrated)

Offset Effects

If one next considers what happens when there is a (DC) offset present in the input signal. If the offset has a positive amplitude k then

$$y(t) = k$$

The indefinite integral of this waveform is given by

$$\int y(t).dt = c + kt$$

which is a ramp of increasing slope proportional to the magnitude (and sign) of k . In the same way that low frequencies can dominate the shape of an integrated waveform, the presence of even a small DC offset can completely alter the structure and magnitude of an integrated signal as seen in the following example.

An engine vibration example

The data in the graph shown in figures 9 is a measured acceleration signal taken from a transducer mounted on an engine. If this signal is integrated without modification then the resultant velocity waveform looks like that in figure 10 with the large increasing ramp

obviously caused by spurious DC or low frequency contributions.

If, however, the acceleration signal is high pass filtered at 5Hz then the resultant waveform still looks essentially the same as can be observed in figure 11. When the filtered signal is integrated, however, the resultant velocity waveform now looks like that in figure 12 and it clearly exhibits a more plausible form of oscillatory behavior.

Choice of high pass cut-off frequency

Commercially available analog vibration integrators typically choose high pass cut-off frequencies of 5Hz for Velocity and 10Hz for Acceleration. This is in the context of an expected analysis range of 10KHz, which means that as a proportion of the working range the filters are set to be less than or equal to 0.05% and 0.01% respectively. With DATS the user has much greater flexibility of choice of cut-off frequency, a fact that becomes increasingly important with signals that have low frequency bandwidths. If possible, it is preferable to choose a low-pass cut-off frequency that is no greater than 50% of the lowest frequency of interest; for example, when analyzing down to 6Hz the cut-off frequency should be set to 3Hz with a cut-off rate of at least 4 passes (8-poles). Normally, filters with Butterworth characteristics are used for both pre and post filtering, but others such as Tchebysheff can also be used although the user should be aware that the cut-off rates vary from filter to filter.

Pre and Post filtering

The concept of post filtering (after integration) was raised in the previous section. This is sometimes necessary to remove residual low frequencies / DC Offsets that sometimes appear after a signal has been integrated. (as in the engine vibration example illustrated above). The standard practice is to apply the same post filter as the pre filter.

Validation of Integration method

In order to verify that the combined filter+integration procedure gives the right result, it is useful to be able to validate the results using a deterministic input signal whose integral can be mathematically predicted. This can be easily carried out using the in-built signal generation facilities within DATS. For example, if one generates a sinusoid of amplitude 3.0, frequency of 10Hz with a sample rate of 1024 sample/sec, when this is integrated it should produce a cosine of amplitude $3.0 / (2 \cdot \pi \cdot 10) = 0.04775$. The corresponding value when calculated from the synthesized waveform

using DATS gives an answer (after stabilization) of 0.04771 which represents an error of less than 0.1%.

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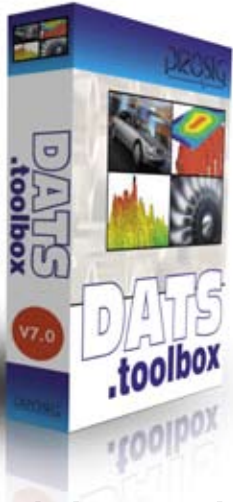
Adrian Lincoln is a director of Prosig Ltd and has responsibilities for signal processing applications, training and overseas sales. He was formerly a Research Fellow at the Institute of Sound & Vibration Research (ISVR) at Southampton University. He is a Chartered Engineer and member of the British Computer Society and Institute of Mechanical Engineers.



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Prosig Software Used in This Article

DATS .toolbox



Acquisition, analysis & reporting

Wide range of analysis functions

**Automated processing through
worksheets & scripts**

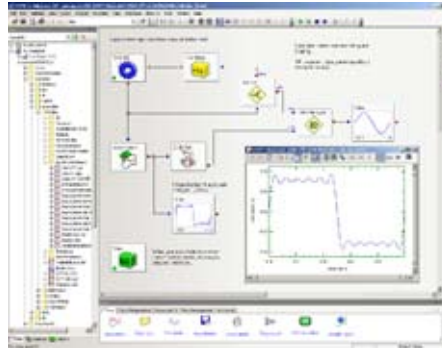
**Built-in report generator
(Intaglio)**

Comprehensive QA features

The DATS software package contains all of the measurement and analysis tools needed in an engineering or research environment. Data acquisition software comes as standard. There is a comprehensive framework of analysis tools (interactive worksheets and scripting). The Intaglio report generator provides template driven, high quality reports.

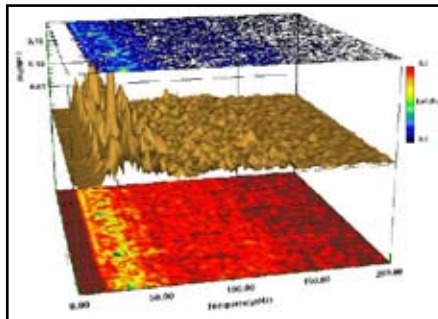
DATS offers outstanding value both in cost and productivity gains. There are no extra costs for data acquisition support, analysis scripting or reporting software. And you won't pay a fortune for support even though we provide a service agreed by many to be the best available. And if you subscribe to our modestly priced support service you will be entitled to regular software updates.

Many man-years of signal processing expertise have been spent on DATS during its 30 years of development. When you purchase DATS you are buying a share in our knowledge. DATS software has proved itself time and time again in diverse and demanding applications around the world.



DATS is built around a fully published, standard data format. A number of features make the DATS data structure unique:

- A complete history of any analyses performed is stored with every signal. This provides a complete audit trail from data capture to final result.
- The Named Element facility provides for storage of everything from data acquisition settings to results of analyses.
- The integrated Project Manager allows a user to configure and store any variables relevant to a test or project with every signal captured or analyzed for that test.

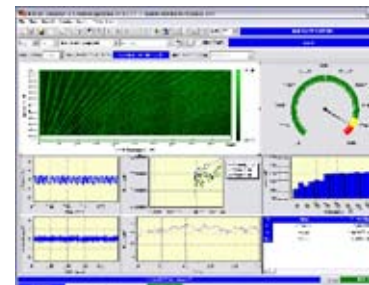


DATS includes a full featured acquisition suite. Capture parameters are easily configured using the setup matrix. During a capture real-time displays show time histories, frequency spectra, numerical levels, gauges, waterfalls and order tracks. Channels can be tagged for immediate

analysis and results of this can be viewed immediately after capture. Automatic averaging of multiple runs is included and averaged runs can be easily reviewed and assessed.

As well as a highly interactive mode, DATS can be automated either using the Visual Scripting environment built into the worksheet interface or using the DATS BASIC scripting language.

Scripts offer a fully featured Visual BASIC® style language. Write your own sequences with data acquisition, input & result forms, DATS analysis functions, reporting, signal access functions, integration with OLE enabled applications (Microsoft® Office etc) and much more.



An important part of most tests or investigations is the final report. DATS includes the Intaglio Report Generator that combines the word processing power of Microsoft® Word and the graphical and analysis capabilities of DATS using OLE technology. Draft reports (templates) are created using Intaglio tools. Each time a report is required the latest data is added to the draft to produce the final report. Reports can be edited by the author, but are "locked" when the report is distributed.

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